Examiners' Report
Principal Examiner Feedback

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Pearson Edexcel International Advanced Level
In Further Pure Mathematics F2 (WFM02)
Paper 01

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Introduction

This paper proved to be a fair test of student knowledge and understanding. There were many accessible marks available to all students as well as some more challenging questions for higher ability students.

Question 1

The opening question on inequalities was a good source of marks for most students. An appropriate algebraic step was required for the first mark and it was rare that this was not achieved. However there were a few cases where the "1" was lost cross-multiplying and errors were sometimes made multiplying both sides by $(x+2)^2$. Those who had obtained a correct equation usually proceeded to achieve the correct critical values. A few did not recognise that $x=-2$ was another critical value. Those who had achieved all three values commonly chose the correct region although a small number gave the region consistent with a < sign in the inequality rather than a > sign. A fairly common error was to see $"-1<x<-2"$. A few gave their answer in set notation – occasionally choosing non-strict inequalities or erroneously using $"\cap"$ to link the two regions. There were also some incidences where the regions $x<2$ and $x>2$ were considered separately but inconsistent critical values were found. It was encouraging to see very few cases of attempts which did not show any use of algebra.

Question 2

This question on complex numbers and de Moivre's theorem also saw good scoring.

In part (a), the correct value for the modulus of z was widely seen and almost all had an appropriate strategy to determine the argument. A small number did not sufficiently identify the $-\frac{\pi}{3}$ as the argument of z. There were a few cases that factorised the given z in $a+\mathrm{i}b$ form or assumed the result and then produced $z=6-6\sqrt{3}\mathrm{i}$ and these were usually correct.

In part (b), most were able to write z in either modulus-argument or exponential form although there were a few cases where "i" was missing and not recovered. De Moivre was usually applied correctly although a few unnecessarily included $+2k\pi$ with their argument. Simplest form was required so the 12^4 had to be evaluated. A few cases were seen where z^4 was attempted by binomial or direct expansion.

In (c), those who had scored in the previous part were usually able to access further marks. A small number divided their argument by $\frac{1}{2}$ rather than 2. A very small number worked with z^4 rather than z. Most went on to achieve a correct root in $a+\mathrm{i}b$ form although it was fairly common for the second root to be omitted. Sometimes conjugate pairs were given as the solutions. A significant number did not use de Moivre and instead formed simultaneous equations from $(a+\mathrm{i}b)^2 = z$. There were occasional slips with this method and as before, sometimes only one root was arrived at.

Question 3

There was generally good scoring with this question on the method of differences.

In part (a), most applied an appropriate multiplier although there were some who were unable to convincingly process the algebra. A few alternatives were seen including making A or r the subject and these were usually correct. This was a “Show that” question and some appropriate supporting working was necessary.

In part (b), the method of differences was recalled and applied by most. A minimum of three appropriate rows needed to be seen although a correct expression was accepted following at least two correct rows. There were some who made errors applying the $"\frac{1}{2}"$ from part (a). Some poor bracketing was seen, as well as some clearly incorrect rows which led to A0 for the final mark.

For those who had obtained the correct expression in (b), full marks in part (c) was fairly common although a significant number lost the $"\frac{1}{2}"$ or were unable to correctly recall the summation formula for integers. A few gave $"n"$ instead of $"\frac{1}{2}n(n+1)"$. A small number lost the last mark with a final answer of $"k = \pm\sqrt{2}"$.

Question 4

Although there were some fully correct responses to this question on Taylor series, there were a wide range of mark profiles awarded. There were several cases where students were clearly uncomfortable differentiating expressions in terms of tan and sec. Those converting to expressions in sin and cos often got into difficulties quickly and did not tend to score well.

In part (a), a correct first derivative was commonly seen. There were a few more errors with determining the second derivative with some unable to recall the correct derivative of $\sec x$. Errors were much more common with the third derivative, with tan squared often seen as just tan. There were also slips handling the "$\frac{3}{2}$" appropriately. This question required all stages of working to be shown and a few students were penalised for finding values of derivatives at $\frac{\pi}{6}$ without finding the derivatives themselves. A small number unnecessarily replaced sec squared with tan squared + 1 which led to mixed results. Most knew the method to produce a Taylor series although there were a few expansions seen in just x and not $x-\frac{\pi}{6}$. A small number used 3 instead of 3! for the coefficient divisor of the fourth term.

In part (b), most did appreciate the need to substitute $\frac{\pi}{4}$ although $\frac{3\pi}{8}$ was sometimes used. Working was not always seen which meant students had to correctly follow through to score the method mark. Care was needed to process the powers of 12 in the denominators but many fully correct answers were seen.

Question 5

This question on an area bounded by a polar curve saw reasonable scoring and quite a lot of fully correct responses. Most were able to apply the area formula although there were a few attempts using r rather than r^2 and the $\frac{1}{2}$ was very occasionally missing. It was disappointing to see quite a few expansions of r^2 where the middle term was lost. The identity that was required to integrate $\cos^2\theta$ was commonly used correctly but many were unable to deal with integrating $\tan^2\theta$, with some attempting this with inappropriate identities (including replacing $\tan^2\theta$ with $1-\sec^2\theta$) or by use of parts which was almost always unsuccessful. Mixed variables were occasionally seen but were usually recovered. Many were able to obtain enough correctly integrated forms to score the integration method mark and a lot of completely correct integrations were seen. Those who had achieved the first seven marks usually went on to score the remaining two although there were a few errors substituting the limits and subtracting, with several adding –20 rather than subtracting it. Occasional errors were seen factorising out the $\frac{1}{12}$.

Question 6

This second order differential equation proved quite challenging for many, with only a small number of fully correct responses seen.

In part (a), most were able to form and solve the auxiliary equation although there were occasional errors processing the result of using the quadratic formula. Some could not produce the correct form for the complementary function. Others were confused by the fact that the question was in terms of x and t rather than y and x. A common error was in selection of the form for the particular integral, with $\lambda t\mathrm{e}^{-3t}$ often chosen. This was often the case with students who were working with a complex function, who perhaps believed the extra t was required since they felt that the correct form of particular integral was already present in their complementary function. A few used $\lambda\mathrm{e}^{-pt}$ but usually recovered the $p=-3$ later. Most used an appropriate method to determine the precise particular integral but full marks was not widely awarded. A small number neglected to include the "$x=...$" as part of their general solution.

Although there were accessible marks in part (b), there were quite a few attempts that made evident confusion with the overall process for solving a second order differential equation. Several worked with just a complementary function (or had one with no constants) or still had an unfound constant in their particular integral. Errors were fairly common obtaining $\frac{\mathrm{d}x}{\mathrm{d}t}$ such as not using the product rule or losing the particular integral. Even some of those who knew the method well were prone to slips with forming and solving the simultaneous equations and so correct particular solutions were not widely achieved.

Part (c) was very discriminating although many found the first mark accessible for setting an expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$ in terms of t equal to zero. However, not many were able to produce a correct trigonometric equation. It was unfortunate to see some students stopping at a value for t rather than continuing to find a value for x (or a). There were also a few slips substituting the value of t back into the particular solution.

Question 7

For those who were familiar with the general method there were a lot of marks scored in this complex number transformation question. However, there were also many confused attempts and very few correct answers to part (b).

In part (a), most knew that making z the subject of the given transformation formula was the sensible way to progress. There were viable alternatives but these were very rarely attempted. There were a surprising number of mistakes achieving the correct expression for z in terms of

w. Most produced an appropriate multiplier that would make the denominator real although, as is normal in this type of question, there were many slips handling the algebra. A recurring error was to omit the denominators of the real and imaginary parts before using $y = x + 3$.The method mark for appropriate progress to manipulate the circle equation was widely scored – those who had the correct circle equation rarely went on to produce an incorrect centre or radius.

The majority were unable to deduce that when mapped by T, the region $y > x + 3$ becomes the interior of the circle. Many shaded the outside or segments of the circle with edges along the axes or sometimes the line $y = x + 3$.

Question 8

The final question on a first order differential equation saw some degree of scoring for most students, although fully correct proofs in (b) and the awarding of the final two marks in (c) was not too common.

In part (a), many students were able to correctly replace $\cot 2x$ and attempt a single fraction. Various identities were then used to make progress. A small number rushed to the given answer without sufficient working. Those using the alternative of writing everything in terms of $\tan x$ tended to be slightly less successful than those following the main scheme. Very few attempts made the poor choice of starting with the right hand side – those that did tended to make little progress. It was pleasing to see a relative absence of “meet in the middle” or “1=1” style proofs.

In part (b) most knew that the sensible strategy was to directly differentiate $y^2 = w\sin 2x$ and many produced the correct result. There were however some poor differentiation attempts and these were much more common where students started by making y or w the subject first. Those who differentiated with respect to w or y often could not achieve an equation in the appropriate derivatives. The third mark for eliminating y was accessible to many but the last mark needed the clear use of the identity proved in part (a) - or suitable equivalent work - and this was not commonly evident.

In part (c), many knew to find an integrating factor and those who had noted the result of the integration of $\text{cosec}\, 2x$ given in the question usually obtained the correct simplified answer. A few used the first result in the formula book which generally led to them being unable to integrate later. The result of integrating $\sec x \tan x$ still remains poorly known and some inefficient attempts at this by parts were seen. However, for those confident with handling trigonometric functions, the first four marks were often scored quite easily. Application of the integrating factor was usually appropriate, although students are advised that it is perfectly acceptable to just write $"w" \times \text{IF} = \int \text{IF} \times "\sec x" \, \text{d}x$ for this step. The last two marks were found to be tougher. The first of these required a correct expression for y^2 following their equation and there were slips with the algebra and manipulation of the trigonometric functions. This mark required a constant of integration which was sometimes missing. Many did go on to achieve an acceptable expression although a small number did not give their answer in terms of $\cos x$ only as requested.